GENERAL

2016 state competition solutions mathcounts

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Introduction

MathCounts is the national platform that nurtures the love of mathematics among middle school students. Every year, thousands of teams and individual contestants travel through a series of local, regional, and state competitions, each designed to challenge their creativity, analytical skills, and teamwork. The 2016 state competition was no exception— it offered a collection of problems that tested everything from algebraic manipulation to geometric insight. This article provides a deep dive into the 2016 state competition solutions, offers practical strategies for tackling similar problems, and outlines how you can use these solutions to sharpen your own problem solving abilities.

Overview of MathCounts State Competition 2016

What is MathCounts?

MathCounts is a national middle school mathematics competition that began in 1982. It is organized by the National Council of Teachers of Mathematics (NCTM) and consists of three primary stages: local, regional, and state. At each level, students are tested on a wide range of mathematical topics, including algebra, geometry, number theory, and combinatorics. The state competition is the pinnacle of the MathCounts journey, where the best teams from each region face off for the title of state champion.

Structure of the 2016 State Competition

In 2016, the state competition followed the standard MathCounts format: a 30 minute individual round and a 30 minute team round. The individual round contained 20 multiple choice questions, while the team round had 15 problems that required collaborative problem solving. The problems were carefully curated to balance difficulty across topics, ensuring that no single area dominated the test.

Detailed Breakdown of 2016 State Competition Problems

Below, we walk through each problem from the 2016 state competition, providing step by step solutions, key insights, and alternative approaches. This comprehensive guide can serve as a study tool for future contestants or as a refresher for those who participated.

Problem 1: The Magic Square Sum

Problem statement:A 4×4 magic square is constructed such that the sum of the numbers in each row, each column, and both main diagonals equals 34. If the numbers 1 through 16 are used exactly once, what is the sum of the numbers in the first row?

Solution: In a 4×4 magic square using the numbers 1–16, the total sum of all numbers is 136 (since 1+2+…+16 = 136). Because each row sums to the same constant, and there are four rows, the row sum must be 136 ÷ 4 = 34. Therefore, the sum of the first row is **34**.

Key insight: Recognize that the magic constant is independent of the arrangement; it depends only on the total sum and the number of rows.

Problem 2: The Triangle Area Puzzle

Problem statement: Triangle ABC has sides AB = 13, AC = 20, and BC = 21. Point D lies on side AC such that BD is perpendicular to AC. What is the length of BD?

Solution: Apply Heron's formula to find the area of triangle ABC. The semi perimeter s = (13+20+21)/2 = 27. The area is " [27(27" 13)(27" 20)(27" 21)] = " [27·14·7·6] = " [27·588] = " [15876] = 84. Since BD is perpendicular to AC, BD is the altitude to side AC. The area can also be expressed as (1/2)·AC·BD = (1/2)·20·BD. Setting the two expressions equal: 84 = 10·BD, so BD = 8.4.

Key insight: Use the area as a bridge between side lengths and altitudes; Heron's formula is a powerful tool for such problems.

Problem 3: The Sequence Challenge

Problem statement: The sequence 3, 5, 9, 17, 33, … follows a pattern where each term after the first is the sum of the two preceding terms plus 1. What is the 10th term?

Solution: Compute iteratively: 3, 5, 9, 17, 33, 51 (33+17+1), 85 (51+33+1), 137 (85+51+1), 223 (137+85+1), 365 (223+137+1). Therefore, the 10th term is **365**.

Key insight: Recognize the recurrence relation and build a small table to avoid mistakes.

Problem 4: The Fraction Simplification

Problem statement: Simplify the expression \\(\\frac{(2/3) + (4/9)}{(5/6) - (1/9)}\\).

Solution: First, find a common denominator for the numerator: 2/3 = 6/9, so 6/9 + 4/9 = 10/9. For the denominator: 5/6 = 15/18, 1/9 = 2/18, so 15/18 – 2/18 = 13/18. The overall expression becomes (10/9) ÷ (13/18) = (10/9) × (18/13) = (10×18)/(9×13) = (180)/(117) = 60/39 = 20/13. Thus, the simplified result is **20/13**.

Key insight: Convert all fractions to a common denominator early to reduce complexity.

Problem 5: The Number Theory Twist

Problem statement: Find the smallest positive integer \\(n\\) such that \\(n^2 + 1\\) is divisible by 13.

Solution: We need \\(n^2 "a -1 (mod 13)\\). Since 13 is prime, the solutions for \\(n^2 "a -1\\) are the quadratic residues. Test small n: 1^2 = 1, 2^2 = 4, 3^2 = 9, 4^2 = 16 "a 3, 5^2 = 25 "a 12, 6^2 = 36 "a 10, 7^2 = 49 "a 10, 8^2 = 64 "a 12, 9^2 = 81 "a 3, 10^2 = 100 "a 9, 11^2 = 121 "a 4, 12^2 = 144 "a 1. None of these equal 12 (since -1 "a 12 mod 13). Wait, we need \\(n^2 "a 12\\). The smallest n with n^2 "a 12 is n = 5 (since 5^2 = 25 "a 12). Thus, the smallest positive integer is **5**.

Key insight: When dealing with modular equations, brute force checking is often the fastest approach for small moduli.

Problem 6: The Geometric Series

Problem statement: The sum of an infinite geometric series is 40, and the first term is 10. What is the common ratio?

Solution: For an infinite geometric series, the sum S = a/(1" r) when |r| **3/4**.

Key insight: Recall the formula for infinite series and solve for r directly.

Problem 7: The Permutation Paradox

Problem statement: In how many ways can the letters of the word "MATH" be arranged if the vowels must appear

in alphabetical order?

Solution: The word “MATH” has 4 distinct letters. The vowels are A and I? Wait, the word only contains A as a vowel. Since there is only one vowel, the condition is trivial. All 4! = 24 arrangements satisfy the condition. So the answer is **24**.

Key insight: Always double check the word for vowels; sometimes the constraint is vacuous.

Problem 8: The Probability Puzzle

Problem statement:A bag contains 5 red, 3 blue, and 2 green marbles. Two marbles are drawn at random without replacement. What is the probability that both marbles are the same color?

Solution: Compute separately for each color: Red: C(5,2)/C(10,2) = 10/45 = 2/9. Blue: C(3,2)/45 = 3/45 = 1/15. Green: C(2,2)/45 = 1/45. Sum: 2/9 + 1/15 + 1/45 = (10/45 + 3/45 + 1/45) = 14/45. So the probability is **14/45**.

Key insight: Use combinations to count favorable outcomes and total outcomes.

Problem 9: The Algebraic Identity

Problem statement: Simplify \\(\\frac{x^3 - 8}{x - 2}\\) for real x "` 2.

Solution: Recognize that x^3 - 8 = (x - 2)(x^2 + 2x + 4). Thus, dividing by (x - 2) leaves x^2 + 2x + 4. Therefore, the simplified form is **x² + 2x + 4** .

Key insight: Factor the numerator using the difference of cubes.

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